NUMERICAL SIMULATION OF THE RELEASED ENERGY IN STRAIN-SOFTENING ROCK MATERIALS AND ITS APPLICATION IN ESTIMATING SEISMIC HAZARDS IN MINES

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This paper describes the use of Rate of Energy Release (RER) to assess the likelihood of mining induced seismic events. We describe a computational framework for simulating RER, the expected and confirmed correlation with induced seismic events and present some example results from mines.

WHAT IS RER?

The mining of excavations in rock re-distributes stress and causes damage to the rock mass and discontinuities. The resulting reduction in strength and degradation in stiffness of the damaged rock and structures leads to further deformation and release of elastic energy. One portion of this released energy is consumed by the damage process - frictional sliding and the creation of new surfaces. This energy cannot be retrieved, so is counted as ‘dissipated’. If the value of the released elastic energy is higher than the energy dissipated by the irreversible damage, the surplus is emitted into the surrounding rock. These release events are seismic events.

The magnitude of the released energy during these events can be measured in a mine using a seismic monitoring system or estimated using a model, as can the instantaneous rate of energy release. The instantaneous, peak rate of energy release from a volume of rock is the Rate of Energy Release (RER).

WHY RER?

In most jurisdictions, tolerable hazard levels for workers are expressed quantitatively. In seismic prone conditions, the engineer must certify that the additional hazard will not cause the total work place risk of injury to be greater than the accepted norm.

This duty requires the engineer to make sufficient, quantitative forecasts of the likelihood, and eventually consequence, of hazardous seismicity. What is the likelihood that a worker will be exposed to some hazard within a certain time frame? Or, how frequently will a worker be within a certain distance x, of an event of magnitude y, within a certain time frame?

At the same time, the mine must be engineered to be productive. Sufficient engineering is an essential constraint, but over-engineering to the point of economic failure is not acceptable. Engineers need quantitative forecasts of seismicity.

Stress, or components of stress are the most common parameters used to estimate seismic hazard levels in mines, but stress is a poor predictor because stress is stored energy. If the energy is not released there are no seismic events. Naturally, stress is an important factor and a suggestive correlation between modelled stress and seismicity is understandable, but such relationships are ‘confounding with causation’.

Measures seeking to quantify the potential for energy release are more reliable. Local Energy Release Density (LERD, after Wiles, 1998), Modelled Ground Work (MGW, after Beck et al 2001), Energy Release Rate (after Ryder, 1987) and nuanced interpretations of the magnitude of induced stress-change all seek to quantify the work that a rock ‘element’ is doing within the load ‘system’. That ‘work’ can correlate with the energy that would be released should that rock be mined or fail.

Board (1996) used the ESS method and a discontinuum model together, to estimate seismic source parameters on fractures around a mining front in a longwall mine, calibrated using measured seismicity. The analysis estimated the likeliest locations and magnitudes for a given mine geometry. Yield (the trigger for energy release) was emergent in the analysis, so the correlations were therefore as mechanically direct as possible.

Beck and Brady (2001) undertook a conceptually similar approach, but attempted to estimate event probability. A cell evaluation technique was used to compare MGW, yield potential and measured events to compute a probabilistic function and measure its predictive efficacy. That analysis showed that if the potential for failure is estimated well enough (for example using ESS), combining that with one of the simple ‘energy’ measures did allow a useable differentiation of high and low risk zones.

Between these 2 studies, limited by the computational power of the day, the ingredients of quantitative seismic forecasting were demonstrated:

1. The occurrence of measured events must be compared to controlling parameters for seismicity to establish a probabilistic forecasting function.
The analysis must account for the geology and nature of discontinuities in the mine.

The controlling parameters for seismicity forecast by the model should be mechanically direct measures of seismic event occurrence and strength, and emergent from the model to facilitate calibration.

The model must simulate the processes that lead to seismic events.

RER can be computed on faults and in the rock mass and can be directly compared to the measure which it seeks to forecast – event occurrence and magnitude. It is also emergent from a strain softening dilatant discontinuum model with an explicit time integration scheme. Computed at points throughout a model of sufficient similitude, calibrated directly using measured seismicity using a scheme such as the Cell Evaluation Method, RER is a good candidate for probabilistic forecasting of seismicity.

**HOW TO COMPUTE RER?**

Simulating RER requires a valid rock mechanics model with an explicit time integration scheme. A valid rock mechanics model for this purpose is one that satisfactorily forecasts both the extent and magnitude of rock mass and discontinuity damage. On this basis, only 3d, dynamic, strain softening, dilatant discontinuum models, able to replicate observed damage and deformation at a sufficient resolution will be applicable for estimating RER.

Few models are built to this specification as they require considerable effort and computing power. So far, all models for RER have employed the Levkovitch Reusch 2 (LR2) discontinuum framework.

**Yield Surfaces (after Levkovitch, Reusch and Beck, 2010) in LR2**

In LR2, a 3d yield surface is used after Menetrey & Willam (1995). It’s parameters allow approximation of all commonly used strength criteria for geomaterials, but accounting for all components of stress. This is an essential element - a 3dimensional yield surface is needed for realistic softening and dilatancy and these are key to simulating damage and RER.

The Menetrey/Willam strength criterion is described by the following function

\[
\left[\frac{q}{\sigma_{ci}}\right]^2 + m \left[\frac{q}{3\sigma_{ci}}\right] R(\theta, e) - \frac{p}{\sigma_{ci}} - s = 0. \tag{1}
\]

The material constants \(s\) and \(m\) are the cohesive and frictional strength, \(\sigma_{ci}\) represents the uniaxial compressive strength, \(p\) is the hydrostatic pressure, \(q\) is the Mises equivalent stress. The dependence on the third invariant is essential for a dilatant model. For LR2 it is introduced via the convex elliptic function in the deviatoric stress plane

\[
R(\theta, e) = \frac{4(1-e^2)\cos^2\theta + (2e-1)^2}{2(1-e^2)\cos\theta + (2e-1)} \tag{2}
\]

The variable \(\theta\), defined via \(\cos 3\theta = (r/q)^3\), is the deviatoric polar angle (also known as Lode angle) and the material constant \(e\) is the deviatoric eccentricity that describes the “out-of-roundedness” of the deviatoric trace of the function \(R(\theta, e)\) in terms of the ratio between the Mises stress along the extension meridian (\(\theta = 0\)) and the compression meridian (\(\theta = \pi/3\)). \(e\) is set to 0.6 to approximate the Hoek Brown yield criterion.

Finally, \(r = \left[9/2 \cdot S \cdot (S \cdot S)\right]^{1/3}\) is the third stress invariant with \(S\) being the deviatoric part of the Cauchy stress \(\sigma\).

**LR2 Plastic strain potential**

The rate of energy release is heavily dependent on load redistribution during yield, so calibrating the plastic strain potential is an essential task. The sufficiency of the plastic strain potential is as fundamental to seismic forecasting as the yield criteria.

The plastic strain potential in LR2 is given by the relation

\[
D_p = \lambda \frac{\partial G}{\partial \sigma} \tag{10}
\]

Where \(\lambda\) the accumulated equivalent plastic strain and \(G\) the flow potential

\[
G = \left[\frac{q}{\sigma_{ci}}\right]^2 + m \left[\frac{q}{3\sigma_{ci}}\right] R(\theta, e) - \frac{p}{\sigma_{ci}} - s. \tag{11}
\]

The model is implemented in such a way that the friction, the cohesion, the dilation and the elastic moduli are prescribed as piecewise linear functions of accumulated plastic strain.

**Structure in LR2**

Discontinuities are such a part of physics of rock, that all rock mechanics phenomena are essentially discontinuity problems. In LR2, faults and shear zones are represented explicitly, free to dislocate, dilate and degrade. Discrete structures that are explicitly represented in the model are modelled with special-purpose interface elements (so-called cohesive elements). These elements may have any valid frictional-cohesive constitutive formulation.

Figure 1 provides an initial guide for representing structures in discontinuum models, after Beck et al 2013 to aide in achieving sufficient similitude for RER computation. Put simply, if the purpose of a model is to estimate behaviour at a certain length scale, how must discontinuities of each length scale range be represented in the model? A line drawn from left to right at the target scale shows the method for incorporating structures of each length scale that line crosses, in that model. The four approaches, usually used in combinations within a single model are explained in the Figure.

When using the guide, the specific circumstances of a modelling task must be accounted for, but the implication for seismic hazard models is that geological structures with persistence from much larger than to smaller than the target
resolution of the model must be represented explicitly in an appropriate manner.

**Strain and damage in LR2**

Damage in LR2 is modelled directly and the whole plastic strain tensor is available to interpret it. Typically, the norm of the deviatoric plastic strain, which is a scalar measure how much plastic strain is accumulated is plotted as this correlates well with most operator's visual interpretation of observed damage. This allows use of a simple colour scale to differentiate meaningful levels of observed damage. LR2s qualitative rock mass scale damage classification for one example is shown in 2 after Beck et al 2009.

The Figure shows simulations of 10m specimens at differing levels of confinement, the damage classification scheme, damage on joints within the simulated specimen and post mining diamond drilled core of locations in a mine where these levels of damage were forecast.

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**Minimum length scales incorporated in model**

<table>
<thead>
<tr>
<th>Target resolution of model</th>
<th>Specimen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tectonic Regional Mineralised System</td>
<td>200000</td>
</tr>
<tr>
<td>Mine Precinct</td>
<td>2000</td>
</tr>
<tr>
<td>Stope/Pillar</td>
<td>250</td>
</tr>
<tr>
<td>Tunnel/ Tunnel Wall</td>
<td>100</td>
</tr>
<tr>
<td>Specimen</td>
<td>50</td>
</tr>
</tbody>
</table>

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**Method of incorporating structure**

<table>
<thead>
<tr>
<th>Method of incorporating structure</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smeared into continuum</td>
<td>Incorporated into the continuum constitutive material model</td>
</tr>
<tr>
<td>Representative fracture network (eg, DFN)</td>
<td>A distribution of explicit discontinuities, in 3d, matching the distribution measured in the real rock mass can be used if there is no explicit structural model for this length scale. A unique, explicit interpretation of discontinuities is preferred</td>
</tr>
<tr>
<td>Unique, Explicit</td>
<td>Faults built in 3d to match the structural geologists interpretation. The 3d geometry of these faults is not substantially simplified</td>
</tr>
<tr>
<td>Effect captured by boundary conditions</td>
<td>Captured only by the effects of these structures on the displacements used to load the boundaries of the model</td>
</tr>
</tbody>
</table>

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**Figure 1** A guide to the method of including discontinuities of different length scales in a geotechnical numerical model, based on the scale of the phenomena that are being targeted after Beck et al 2013
The models show the relation between discontinuity damage and stages of material instability and rock mass degradation. This is fundamental to understanding how RER and seismicity evolve.

During the elastic stage (I), there is virtually no new damage on the joints; there are only acoustic emissions and there is no substantive degradation in rock mass properties. At slightly higher loads, during initial quasi-stable plastic degradation (II), new damage on some pre-existing discontinuities nucleates. There is some degradation in stiffness at the rock mass scale, but additional stress is needed still to continue deformation - the rock mass softens without becoming materially unstable. At higher strains again, an unstable phase (III) develops. Mechanically, additional discontinuity sets are damaged, damage between adjacent nucleation sites coalesces, and the structural integrity of the specimen diminishes. This behaviour is not occurring by definition of the constitutive model alone; it is manifesting as a result of the combined effects and balance reached between of stress, strain strength and structure.

At each of these stages, energy is released and RER can be computed from an LR2 model. The relation between strain and simulated acoustic emissions is shown on the figure:

- Acoustic emission commences pre-peak.
- There is a rapid rise in acoustic emission rate during what would be interpreted as the transition to initial quasi-stable plastic degradation (II in the Figure).
- The rate of acoustic emissions decreases during the transition to unstable deformation (III), but there are sometimes some larger events (see Beck et al, 2009) and a step change in deformation in the specimens. For more brittle rocks this transition occurs more quickly and events were larger, as would be expected.
- After Stage III there is a rapid decrease in acoustic emissions.

Importantly, the specific mechanisms of pre-peak seismicity, seismogenic specimen instability and eventual near-aseismic comminution are visible in the DFN tests.

It should be clear that softening, dilatancy and discontinuities are critical elements of realistic RER. For this reason, a main effort of model calibration should be to match the softening and dilatancy response of the rock and discontinuities.

Figure 2  Comparison of stress-strain-energy release of a simulated rock mass specimen (8m diameter) at varying levels of confinement and photographs of diamond drill core specimens, from locations estimated by calibrated global models to be at these respective states of strain, after Beck et al 2009.

![Figure 2](image_url)
Event Probability and RER

To establish a quantitative relation between modelled RER and expected or measured seismic potential, the ‘Cell Evaluation Method’ or CEM (Beck & Brady, 2001) is employed. In the current context, RER is compared to seismic event occurrence using the CEM as a statistical tool.

First, the entire model is discretised into regular, volumetric ‘cells’ or ‘test blocks’. RER is calculated in every single test block within the model, for every single model step. Recall that RER is the highest rate of energy release in the model during that increment of time.

Next, the number of real, measured seismic events in each test block is counted for each and every model step. When the data is assembled, the probabilistic correlation between real event and modelled RER can be computed. The relation between RER and the event probability, \( p(X) \) of a mine tremor of a certain magnitude, \( X \), occurring in a test block, is denoted:

\[
p(X) = x \approx \frac{n_{RER}^{i}}{e_{RER}^{j}}
\]  

[12]

Where the total number of test blocks having values within any range of RER release rate is denoted \( e_{RER}^{j} \), where \( j \) is the fixed interval of RER being evaluated and the sum of blocks containing events within that magnitude range as \( n_{RER}^{i} \), where \( i \) is the event magnitude range (eg 0ML to 1ML) being considered.

There are a number of conditions that have to be satisfied for \( p(X) \) to be a true probability, and these are outlined in Beck and Brady 2001.

An example correlation between RER and measured event occurrence at a mine is shown in Figure 3. The correlation is based on over 30000 events measured over 4 years at a deep block caving mine. It shows a very clear, exponential relationship between RER and event occurrence. The relationship and how this can be used to calibrate a model can be understood by considering the relation between damage and acoustic emissions, after Beck et al, 2009.

Acoustic emissions were indicated in that numerical experiment by high frequency, transient stress waves measured at select locations within simulated discontinuous FE specimens. The complete velocity record of these nodes was retained and back analysed to produce this plot (see Beck et al, 2009 for more details).

A global scale example was outlined in Beck, Levkovitch and Simser (2012), for the Nickel Rim South Mine in Canada. The model incorporated stop e by stope extraction, a very detailed structural model and a high resolution geological model. LR2 was implemented as described above, and examples of the close correlation between modelled RER and seismicity are shown in Error! Reference source not found.. The figure shows measured event clusters as wireframes, and various measures of modelled energy release as volume rendered clouds.

The event clusters are inter-event (IE) distance isosurfaces of approximately 5-10m. For this study, high intensity seismic activity was defined as a cluster with an IE distance of <5m. Moderate intensity corresponded to an IE distance of <10, seismogenic zones i.e. continuous and bounded regions of increased activity above random, or background microseismicity was assessed by volumes with an IE distance of <25m. Areas with an IE distance of >25m were considered to experience low level or background noise only.
The effect of the process is to express exposure of each area of the mine to seismicity as a function of events of a certain magnitude, within a certain distance, during an explicit period, as required.

An example from another mine is shown in Figure 5, after Beck and Putzar 2011. It shows an example match between modeled RER and measured seismic events. The close match is representative of the models performance during each month of the study period. Ultimately, the close match between the forecasts and measured data validated the tool for its intended use, assisting the mine to plan extraction strategies.
CONCLUSIONS

The discontinuum LR2 framework allows calibrated global scale assessments of induced deformation and seismic potential using Rate of Energy Release (RER). The reasons why the model can match the measured seismicity sufficiently are the careful matching of the actual and modelled extraction sequencing, the large strain, strain softening constitutive formulation, the ability to match realistic softening across length scales, the incorporation of a very large and sufficient number of explicit structures and practically, a high standard for quantitative, direct (as close as possible, like for like) calibration.

The tool does not replace any other component of sound mine design, day-to-day geotechnical practice or seismic risk management; rather it is an adjunct to existing mine design procedures to minimise seismic hazards.

REFERENCES


Reusch, F., Levkovitch, V. & Beck, D. 2010: "Multi-scale, non-linear numerical analysis of mining induced deformation in complex environments" in "Rock Mechanics in Civil and Environmental Engineering" , Jian Zhao (Editor), Vincent Labiouse (Editor), Jean-Paul Dudt (Editor), Jean-Francois Mathier (Editor), CRC Press, 2010, 697-700.
